

Chapter 9 / **Example 13****Diagonalisation and powers of a matrix**

- a** Find the diagonalization of  $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$
- b** Hence find an expression for  $A^4$  in the form  $PD^4P^{-1}$ .
- c** Find an expression for  $A^4$  as a product of 3 matrices with no exponents.

$$|A - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - \lambda - 6 = 0$$

To solve the quadratic equation press **[APPS]** :PlySmlt2

Press **[ENTER]** and select 1:POLYNOMIAL ROOT FINDER

You are solving an equation of order 2

Press **[F5]** NEXT.

```

POLY ROOT FINDER MODE
ORDER  1 2 3 4 5 6 7 8 9 10
REAL   a+bi  re^(θi)
DEC    FRAC
NORMAL SCI  ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
[MAIN] [HELP] [NEXT]

```

Enter the coefficients of the quadratic equation.

```

a2x2+a1x+a0=0
a2=1
a1=-1
a0=-6
[MAIN] [MODE] [CLEAR] [LOAD] [SOLVE]

```

Press **[F5]** SOLVE.

The calculator displays the solution  $\lambda_1 = 3, \lambda_2 = -2$

The eigenvectors are

$$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } x_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Press **[F1]** MAIN 6:QUIT APP

```

a2x2+a1x+a0=0
x1=3
x2=-2
[MAIN] [MODE] [COEFF] [STORE] [F<=>D]

```

$$P = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix}$$

Press **[2nd]** **[x<sup>-1</sup>]** **[MATRX]** **[▶]** **[▶]** EDIT 1:[A] and press **[ENTER]**.

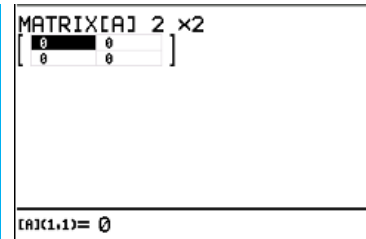
```

NAMES MATH EDIT
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7:[G]
8:[H]
9:[I]

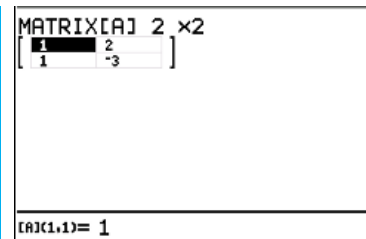
```

Chapter 9 / **Example 13****Diagonalisation and powers of a matrix**

Change the dimensions of the matrix to  $2 \times 2$  and press **ENTER**.



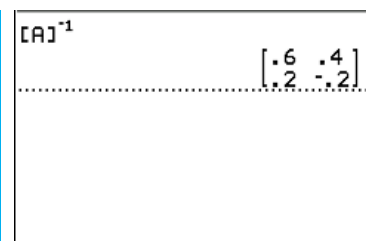
Enter the values of the elements of the matrix  $P$ , using **ENTER** to move through the matrix.



Press **2nd** **MODE** **[QUIT]**.

Press **2nd**  **$x^{-1}$**  **[MATRX]** 1:[A] and press **ENTER**.

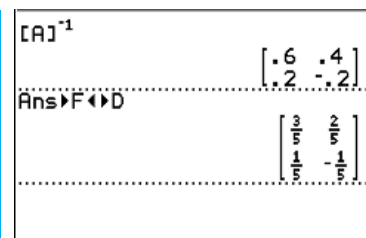
Press  **$x^{-1}$**  and press **ENTER**.



Press **[ALPHA]** **[F1]** 4:►F◀D and press **ENTER**.

The GDC displays the matrix  $P^{-1}$  in fractional form.

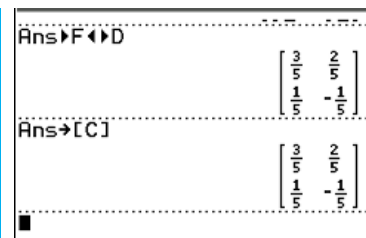
$$P^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$



Store the result as C.

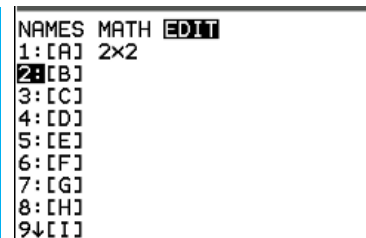
Press **STO►**.

Press **2nd**  **$x^{-1}$**  **[MATRX]** 3:[C] and press **ENTER**.



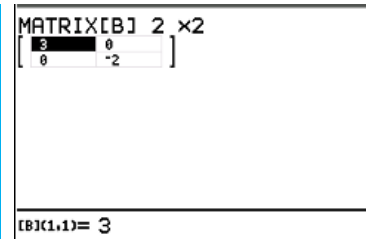
$$D = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

Press **2nd**  **$x^{-1}$**  **[MATRX]** ► ► EDIT 2:[B] and press **ENTER**.



Chapter 9 / **Example 13****Diagonalisation and powers of a matrix**

Change the dimensions of the matrix to  $2 \times 2$  and press **ENTER**.  
Enter the values of the elements of the matrix  $D$ .



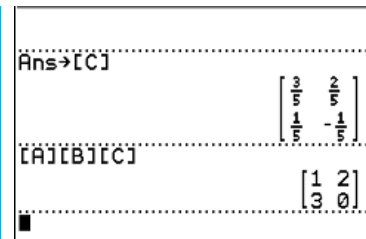
$$A = PDP^{-1}$$

Press **2nd** **MODE** **[QUIT]**.

Press **2nd**  **$x^{-1}$**  **[MATRX]** 1:[A]

Press **2nd**  **$x^{-1}$**  **[MATRX]** 2:[B]

Press **2nd**  **$x^{-1}$**  **[MATRX]** 3:[C] and press **ENTER**.



$$A^4 = PD^4P^{-1}$$

Press **2nd**  **$x^{-1}$**  **[MATRX]** 2:[B] and press  **$\wedge$**  and type 4

$$D^4 = \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$

